



## Computational Mathematics Day at UTD:

### Numerical Methods and Uncertainty Quantification for Porous Media Flows

Tuesday, MAY/08, 2018 – Room FO 2.404

#### Program:

**9:55 am – 10:00 am:** Welcome remarks. Felipe Pereira, UTD

**10:00 am – 10:40 am:** *A multiscale method with Robin boundary conditions for the Darcy flow problem.* Rafael T. Guiraldello, University of São Paulo (USP), Brazil

**10:50 am – 11:30 am:** *A new downscaling procedure for multiscale methods in porous media flows.* Fabrício S. Sousa, University of São Paulo (USP), Brazil

**11:40 am – 12:20 pm:** *A recursive parallel implementation of the Multiscale Mixed Method.* Paola Ferraz, University of Campinas (UNICAMP), Brazil

**Lunch: 12:30 pm – 2:00 pm**

**2:00 pm – 2:40 pm:** *A computational method for solving a two-phase Buckley-Leverett pseudo-parabolic model with dynamic capillary pressure.* Eduardo Abreu, University of Campinas (UNICAMP), Brazil

**2:50 pm – 3:30 pm:** *Markov chain Monte Carlo methods (McMC) applied to deformable porous media flows.* M. R. Borges, National Laboratory for Scientific Computing (LNCC), Brazil

**3:40 pm – 4:20 pm:** *Markov chain Monte Carlo methods applied to subsurface flow problems: a convergence study.* Abdullah Al Mamun, UTD

**4:30 pm – 5:10 pm:** Application of ensemble-based methods to history matching. Samson Folarin, UTD

## Abstracts:

### **TITLE: A multiscale method with Robin boundary conditions for the Darcy flow problem**

Speaker: Rafael T. Guiraldello

The design of accurate multiscale domain decomposition methods for channelized, high-contrast porous media remains as an important challenge for the numerical simulation of typical problems posed by the oil industry. Here we investigate an improved version of the recently proposed Multiscale Robin Coupled method (MRCM) [1], which is a generalization of the Multiscale Mixed Method (MuMM) [2]. This method ensures weak continuity of both normal fluxes and pressure through the imposition of Robin-type boundary conditions at the skeleton of the domain decomposition where interface spaces for the pressure and fluxes, respectively, can be chosen independently. Numerical simulations are presented aiming at illustrating several features of the MRCM. We show that the MRCM can also be seen as a generalization of two well known multiscale procedures, the Multiscale Mixed Mortar Finite Element method (MMMFEM) [3] and the Multiscale Hybrid Method (MHM) [4]. Then, we compare the accuracy of the above mentioned multiscale procedures given a fixed computational cost for interface spaces spanned by polynomial and informed functions [5]. Our results illustrate how one can take advantage of the built in flexibility of the MRCM to produce more accurate approximations when compared to the MMMFEM and MHM.

[1] R.T. Guiraldello, R.F. Ausas, F.S. Sousa, F. Pereira, G.C. Buscaglia, The Multiscale Robin Coupled Method for flows in porous media, In *J. Comput. Phys.* 355 (2018) 1-21.

[2] A. Francisco, V. Ginting, F. Pereira, J. Rigelo. Design and implementation of a multiscale mixed method based on a nonoverlapping domain decomposition procedure, *Math. Comput Simul.*, 99:125–138, 2014.

[3] T. Arbogast, G. Pencheva, M.F. Wheeler, I. Yotov, A multiscale mortar mixed finite element method, *SIAM Multiscale Model. Simul.* 6 (1) (2007) 319–346.

[4] R. Araya, C. Harder, D. Paredes, F. Valentin, Multiscale hybrid-mixed method, *SIAM J. Numer. Anal.* 51 (6) (2013) 3505–3531.

[5] E. Chung, S. Fu, Y. Yang, An enriched multiscale mortar space for high contrast flow problems, arXiv:1609.02610, 2016.

### **TITLE: A new downscaling procedure for multiscale methods in porous media flows**

Speaker: Fabricio S. Sousa

Non-overlapping domain decomposition multiscale methods have been successfully applied to flows in porous media. Such remarkable class of methods seek to decompose the domain of the porous media flow equations in non-overlapping subdomains, solving smaller local problems in parallel, and one global interface problem, instead of a large coupled one. Usually the interface problem enforces the compatibility conditions -- continuity of pressures and normal fluxes -- across subdomain interfaces. In multiscale methods, these conditions are enforced at different length scales, originating different methods. Examples are the MMMFEM [1] (Multiscale Mixed Mortar Finite Element Method), that prioritizes pressure continuity at fine scales and weak flux continuity; the MHM [2] (Multiscale Hybrid-Mixed Method), that enforces continuity of normal fluxes at fine scales and weak pressure continuity; the [3] (Multiscale Mixed Method) that enforces weak continuity of both pressure and normal fluxes through Robin-type boundary conditions. Recently, the MRCM [4] (Multiscale Robin Coupled Method) that generalizes the aforementioned methods in one single variational formulation has been introduced.

It is well known that enforcing continuity only at larger scales creates a discrepancy in the continuity (of pressure or normal fluxes, or both) in fine scales, so that a downscaling is required to keep the resulting velocity fields conservative.

We propose a new procedure based on alternating domains with minimum overlapping to perform downscaling of the computed normal fluxes, resulting in new conservative velocity fields. We studied the applicability and efficiency of this new method when applied to single-phase flow problems using the MRCM, as compared to existing techniques, exploring the choice of parameters given by the MRCM to improve its accuracy.

- [1] T. Arbogast, G. Pencheva, M.F. Wheeler, I. Yotov, A multiscale mortar mixed finite element method, *SIAM Multiscale Model. Simul.* 6 (1) (2007) 319–346.
- [2] R. Araya, C. Harder, D. Paredes, F. Valentin, Multiscale hybrid-mixed method, *SIAM J. Numer. Anal.* 51 (6) (2013) 3505–3531.
- [3] A. Francisco, V. Ginting, F. Pereira, J. Rigelo, (2014). Design and implementation of a multiscale mixed method based on a non-overlapping domain decomposition procedure. *Math. Comput. Simul.*, 99, 125-138.
- [4] R.T. Guiraldello, R.F. Ausas, F.S. Sousa, F. Pereira, G.C. Buscaglia, The Multiscale Robin Coupled Method for flows in porous media, In *J. Comput. Phys.* 355 (2018) 1-21.

**TITLE: A Recursive Parallel Implementation of the Multiscale Mixed Method**

Speaker: Paola Ferraz

We are interested in the numerical approximation of partial differential equations of elliptic-hyperbolic nature, in the context of incompressible two-phase flow problems in heterogeneous porous media. Numerical solutions of elliptic boundary value problems with high contrast and discontinuous coefficients are often expensive and time consuming, so efficient numerical methods are necessary. Indeed, methods that can take advantage of CPU-GPU clusters are of particular interest because GPUs have larger computational power than CPUs alone. In this work, we focus on the multiscale mixed method MuMM introduced in [1] (see also [2] where the variational formulation of [1] was presented), that is based on a non-overlapping iterative domain decomposition procedure with Robin interface conditions. Local multiscale basis functions are calculated in each subdomain to represent the discrete solutions that can be efficiently computed in CPU-GPU clusters. The method presented here uses a new technique to cluster multiscale basis functions associated with nearest neighbor subdomains, leading to a small (and local) linear system for the interface between the subdomains. The global solution is obtained by recursively applying the MuMM to all pairs of subdomains until the union of subdomains reach the whole domain. The resulting interface linear systems are easily handled by Schur decomposition along with a LU factorization. The novelty of this method is that it does not use an iterative procedure to compute the global solution and shows excellent parallel performance.

- [1] A. Francisco, V. Ginting, F. Pereira and J. Rigelo, (2014). "Design and implementation of a multiscale mixed method based on a nonoverlapping domain decomposition procedure", *Math. Comput. Simul.*, 99, 125-138.
- [2] R.T. Guiraldello, R.F. Ausas, F.S. Sousa, F. Pereira and G.C. Buscaglia, (2018). "The Multiscale Robin Coupled Method for flows in porous media", *Journal of Computational Physics*, 355, 1-21.

**TITLE: A computational method for solving a two-phase Buckley-Leverett pseudo-parabolic model with dynamic capillary pressure**

Speaker: Eduardo Abreu

We formally discuss a numerical approach for the simulation of a two-dimensional model for immiscible incompressible two-phase flows in heterogeneous porous media to deal with dynamic capillary pressure. The differential system of equations consist of a nonlinear pseudo-parabolic Buckley-Leverett equation modeling the transport of fluid saturations coupled with an elliptic equation modeling the Darcy pressure-

velocity problem. Upon certain manipulation of the differential system of equations, we rewrite the transport pseudo-parabolic Buckley-Leverett model as a nonlinear elliptic reaction-diffusion problem along with a simpler time-dependent relation to deal with the dynamic capillary pressure. Hybridized mixed finite elements and domain decomposition procedures are used for the spatial discretization of the equations. Proper locally conservative finite volume techniques are used for a balancing discretization of the first-order hyperbolic flux and the dispersive behavior inherent of the pseudo-parabolic model. After discretizing in time, the resulting nonlinear algebraic equations is solved with a sequential time-marching approach and the arising systems of linear equations are solved efficiently. Our numerical experiments demonstrated the viability of the proposed formulation based on 1D and 2D computational simulations for porous media nonlinear flows. Joint work with Paola Ferraz (University of Campinas) and Jardel Vieira (University of Campinas).

**TITLE: Markov chain Monte Carlo methods (McMC) applied to deformable porous media flows**

Speaker: M. R. Borges

Natural reservoirs exhibit high degree of spatial variability in their properties in multiple length scales. It has been established that such variability has a strong impact in determining fluid flow patterns in subsurface formations. Direct measurements of reservoir properties are only available at a small number of locations. Without an adequate description of the formation properties, such as hydraulic conductivity, porosity, and poromechanical parameters, the predictability of computational models is limited and tends to deteriorate over time. In this sense, history matching and uncertainty quantification are important research topics that aim at reducing the uncertainty in reservoir performance forecasting. Dynamic flow data are direct measures of reservoir responses and can introduce important information about subsurface processes. However, due to the non-linearity of the relations between flow data and reservoir properties, the integration of dynamic flow data directly in computational models by conventional geostatistics techniques is usually difficult. The problem of matching dynamic data can be formalized, as a inverse problem, in terms of Bayesian analysis and Markov chain Monte Carlo (McMC) methods. The Bayesian framework allows quantifying the added value of information from several sources, while McMC methods allows sampling from the *posterior* distribution in a computational framework. In the Bayesian formulation, a *prior* distribution of geologically consistent reservoir parameters must be assumed from *prior* geological information or generated using information from geostatistical analysis conducted on static data from core samples. Recently, the role of geomechanics in subsurface petroleum reservoirs and freshwater-bearing formations is becoming increasingly important as deeper formations are detected and explored. Accurate predictions of fluid injection in weak rocks require a detailed coupled flow simulation and mechanical deformation modeling and demand precise understanding of the geomechanical factors affecting the hydrodynamics. During the exploration of a subsurface formation, changes in pore pressure trigger perturbations in the mechanical equilibrium of the porous medium leading to stress modifications, which alter rock properties such as permeability and porosity. In this work, we use the Markov-chain Monte Carlo (McMC) method to characterize the permeability, porosity and Young's modulus fields in a two-phase flow problem coupled with the geomechanics of the adjacent rocks.

**TITLE: Markov Chain Monte Carlo Methods Applied to Subsurface Flow Problems: A Convergence Study**

Speaker: Abdullah Al Mamun

In subsurface characterization using a history matching algorithm subsurface properties are reconstructed

with a set of limited measured data. Here we focus on the characterization of the permeability field in an aquifer using Markov chain Monte Carlo algorithms, which are reliable procedures for such reconstruction. The MCMC method is serial in nature due to its Markov property. Moreover, the calculation of the likelihood information in the MCMC is computationally expensive for subsurface flow problems. Running a long MCMC chain for a very long period makes the method less attractive for the characterization of the subsurface. In contrast, several shorter MCMC chains can substantially reduce computation time and can make the framework more suitable to subsurface flows. However, the convergence of such MCMC chains should be carefully studied. In this presentation we consider multi-MCMC chains for a single-phase flow problem and analyze the chains aiming at a reliable characterization. We consider both single-stage and two-stage MCMC algorithms in our study.

**TITLE: Application of Ensemble-Based methods to reservoir simulation**

Speaker: Samson Folarin

In reservoir simulation ensemble-based inverse methods have been widely used to quantify the uncertainties in subsurface porous media flows. In this presentation we will provide an introduction to such methods. Our discussion will focus on the Ensemble Kalman Filter [1], the Half Iterative Ensemble Kalman Filter [2] and Ensemble Smoother [3]. Advantages and drawbacks of these procedures will be pointed out.

[1]- Tavakoli R., Pencheva G., Wheeler M. F., Ganis B.: A parallel ensemble-based framework for reservoir history matching and uncertainty characterization. *Comput Geosci* (2013) 17:83-97.

[2]- Yudou W., Maohui L.: Reservoir history matching and inversion using an iterative ensemble kalman filter with covariance localization. *Pet.Sci* (2011) 8:316-327.

[3]- Emerick A. A. , Reynolds A. C.: Ensemble Smoother with Multiple Data Assimilation. *Computers and Geosciences* 55 (2013) 3-15.